Asynchronous development of the Benjamin-Feir unstable mode: Solution of the Davey-Stewartson equation

Masayoshi Tajiri, Kiyohiro Takeuchi, and Takahito Arai
Department of Mathematical Sciences, Graduate School of Engineering, Osaka Prefecture University, Sakai, Osaka 599-8531, Japan
(Received 3 January 2001; published 29 October 2001)

The long time evolution of the Benjamin-Feir unstable mode in two dimension is described by the growing-and-decaying mode solution to the Davey-Stewartson equation. The solution of the hyperbolic Davey-Stewartson (the so-called Davey-Stewartson I) equation is analyzed to show that the resonance between line soliton and growing-and-decaying mode exists. If the resonant condition is exactly satisfied, the growing-and-decaying mode exists only in the forward region of propagation of soliton and the soliton is accelerated (or decelerated). Under the quasiresonant condition, the growing-and-decaying mode grows at first in the forward region, and after the sequence of the evolution has done in the forward region the mode starts to grow in the backward region of the soliton.

DOI: 10.1103/PhysRevE.64.056622 PACS number(s): 05.45.Yv

I. INTRODUCTION

A weakly nonlinear, uniform, water wave train is unstable to long wave modulational perturbations of the envelope, which is known as the Benjamin-Feir (B-F) instability [1,2]. The long time evolution of a two-dimensional wave packet is described by the Davey-Stewartson (DS) equation [3–6]. One of the important features of the soliton of the DS equation is the reverting of the unstable wave train to its initial state, the so-called Fermi-Pasta-Ulam recurrence. It is known that the nonlinear evolution of the unstable mode is described by the growing-and-decaying mode solution [7].

Recently, the interactions between two-periodic solitons, between the periodic soliton and line soliton and between the periodic soliton and algebraic soliton to the DS equation have been investigated in detail [7–11]. It is shown that the periodic soliton resonances exist in each case. Pelinovsky pointed out the existence of the resonance between line soliton and growing-and-decaying mode [8]. He has shown that a new line soliton occurring as a result of the Benjamin-Feir instability moves in the opposite direction with respect to the original soliton. The growing-and-decaying mode solution grows exponentially according to the linear instability at the initial stage, which reaches the maximum amplitude after some finite time and then damps out at a sufficient large time. The growing-and-decaying mode exists substantially during only a finite period in time, but the resonance between line soliton and growing-and-decaying mode brings an infinite phase shift to the line soliton phase. If the growing-and-decaying mode exists within only a finite time in reality, we are very interested in the mechanism that brings about the infinite phase shift of the line soliton.

In this paper, the solution to the DS equation is analyzed to investigate the time evolution of the resonant interaction between line soliton and the growing-and-decaying mode. It is shown that the existence of soliton changes the evolution of the B-F instability drastically.

II. RESONANCE BETWEEN LINE SOLITON AND GROWING-AND-DECAYING MODE

The DS equation may be written as

$$\begin{cases} iu_t + pu_{xx} + u_{yy} + r|u|^2u - 2uv = 0, \\ v_{xx} - pv_{yy} - r(|u|^2)_{xx} = 0, \end{cases} \tag{1}$$

where $p = \pm 1$, r is constant. Equation (1) with $p=1$ and $p=-1$ are called the DS I and DS II equations, respectively. The interaction between line soliton and growing-and-decaying mode with respect to the DS I equation is studied in this paper. The solution describing the interaction can be obtained by the N-soliton solution of Satsuma and Ablowitz [12].

The three-soliton solution to the DS I equation may be written as [12]

$$u = u_0 e^{i\zeta}\frac{g}{f}, \quad v = -2(\ln f)_{xx} \tag{2}$$

with

$$\begin{aligned} f = {} & 1 + e^{\eta_1} + e^{\eta_2} + e^{\eta_3} + a(1,2)e^{\eta_1+\eta_2} + a(1,3)e^{\eta_1+\eta_3} \\ & + a(2,3)e^{\eta_2+\eta_3} + A\, e^{\eta_1+\eta_2+\eta_3}, \end{aligned} \tag{3}$$

$$\begin{aligned} g = {} & 1 + e^{\eta_1 + i\phi_1} + e^{\eta_2 + i\phi_2} + e^{\eta_3 + i\phi_3} + a(1,2) \\ & \times \exp[\eta_1 + \eta_2 + i(\phi_1 + \phi_2)] + a(1,3) \\ & \times \exp[\eta_1 + \eta_3 + i(\phi_1 + \phi_3)] \\ & + a(2,3)\exp[\eta_2 + \eta_3 + i(\phi_2 + \phi_3)] \\ & + A\exp[\eta_1 + \eta_2 + \eta_3 + i(\phi_1 + \phi_2 + \phi_3)], \end{aligned} \tag{4}$$

where

$$\zeta = kx + ly - \omega t, \tag{5}$$

$$\eta_j = K_j x + L_j y - \Omega_j t + \eta_j^0, \tag{6}$$

$$\omega = k^2 + l^2 - ru_0^2, \tag{7}$$

$$\sin^2\frac{\phi_j}{2} = \frac{K_j^2 - L_j^2}{2ru_0^2}, \tag{8}$$

$$\Omega_j = 2kK_j + 2lL_j - (K_j^2 + L_j^2)\cot\frac{\phi_j}{2}, \tag{9}$$

$$a(i,j) = \frac{2ru_0^2 \sin\frac{\phi_i}{2}\sin\frac{\phi_j}{2}\cos\frac{\phi_i - \phi_j}{2} - K_iK_j + L_iL_j}{2ru_0^2 \sin\frac{\phi_i}{2}\sin\frac{\phi_j}{2}\cos\frac{\phi_i + \phi_j}{2} - K_iK_j + L_iL_j}, \tag{10}$$

$$A = a(1,2)a(1,3)a(2,3), \tag{11}$$

The growing-and-decaying mode solution to the DS I equation is given by [7]

$$u = u_0 e^{i(\zeta + \phi_1)} \frac{\sqrt{M}\cosh(\Omega t + \sigma_1 - i\phi_1) - \cos\eta}{\sqrt{M}\cosh(\Omega t + \sigma_1) - \cos\eta}, \tag{12}$$

$$v = -2\beta^2 \frac{\sqrt{M}\cosh(\Omega t + \sigma_1)\cos\eta - 1}{\{\sqrt{M}\cosh(\Omega t + \sigma_1) - \cos\eta\}^2}, \tag{13}$$

where

$$\eta = \beta x + \delta y - \gamma t + \theta, \tag{14}$$

$$\Omega = (\beta^2 + \delta^2)\cot\frac{\phi_1}{2}, \tag{15}$$

$$\gamma = 2k\beta + 2l\delta, \tag{16}$$

$$M = a(1,2) = \frac{2}{1 + \cos\phi_1} > 1, \tag{17}$$

$$\sin^2\frac{\phi_1}{2} = \frac{\delta^2 - \beta^2}{2ru_0^2}, \tag{18}$$

σ_1 and θ are arbitrary phase constants, which are obtained by putting the x and y components of wave numbers, K_j and L_j $(j=1,2)$ in the two-soliton solution as follows: $K_1 = K_2^* = i\beta$, $L_1 = L_2^* = i\delta$. The existence condition for the nonsingular solution is given by $M > 1$ for real ϕ_1. This solution grows exponentially at initial stage and reaches a state of maximum modulation and after reaching maximum modulation, demodulates and finally returns to an unmodulated state.

It is well known that the line soliton solution can be obtained by putting the x and y components of wave number real constants in one-soliton solution. By putting the x and y components of wave numbers in the three-soliton solution in Eqs. (3) and (4) as,

$$K_1 = K_2^* = i\beta, \quad L_1 = L_2^* = i\delta, \quad K_3 = P, \quad L_3 = Q, \tag{19}$$

we have the solution consisting of a line soliton and growing-and-decaying mode,

$$u = u_0 e^{i\zeta}\frac{g}{f}, \quad v = -2(\ln f)_{xx} \tag{20}$$

with

$$f = 1 + e^{-\Omega t + \sigma_1}\cos\eta + \frac{M}{4}e^{-2\Omega t + 2\sigma_1} + e^{\xi}\left[1 + e^{-\Omega t + \sigma_1}\{N_r\cos\eta - N_i\sin\eta\} + \frac{M|N|^2}{4}e^{-2\Omega t + 2\sigma_1}\right], \tag{21}$$

$$g = 1 + \exp(-\Omega t + \sigma_1 + i\phi_1)\cos\eta + \frac{M}{4}\exp(-2\Omega t + 2\sigma_1 + 2i\phi_1) + e^{\xi + i\phi_2} \times\left[1 + \exp(-\Omega t + \sigma_1 + i\phi_1) \times\{N_r\cos\eta - N_i\sin\eta\} + \frac{M|N|^2}{4} \times\exp(-2\Omega t + 2\sigma_1 + 2i\phi_1)\right], \tag{22}$$

where

$$\xi = Px + Qy - \Gamma t + \sigma_2, \tag{23}$$

$$\sin^2\frac{\phi_2}{2} = \frac{P^2 - Q^2}{2ru_0^2}, \tag{24}$$

$$\Gamma = 2kP + 2lQ - (P^2 + Q^2)\cot\frac{\phi_2}{2}, \tag{25}$$

$$N = N_r + iN_i = \frac{2ru_0^2 \sin\frac{\phi_1}{2}\sin\frac{\phi_2}{2}\cos\frac{\phi_1 - \phi_2}{2} - i(\beta P - \delta Q)}{2ru_0^2 \sin\frac{\phi_1}{2}\sin\frac{\phi_2}{2}\cos\frac{\phi_1 + \phi_2}{2} - i(\beta P - \delta Q)}, \tag{26}$$

σ_2 is an arbitrary constant.

The phase shift of the line soliton due to the growing-and-decaying mode is given by the amount $\Psi = \ln|N|$ (or $-\ln|N|$). $\Psi = \infty$ and 0 may be thought of as resonance between line soliton and growing-and-decaying mode, the conditions of which are obtained by equating the denominator and numerator of N to zero, respectively:

TABLE I. The classification of the resonant interaction by (ϕ_1,ϕ_2). The sign of $\tilde{\Gamma}$ $[\tilde{\Gamma}=\Gamma-2(kP+lQ)=-(P^2+Q^2)\cot(\phi_2/2)]$ specifies the time evolution: for the case $\tilde{\Gamma}<0$, the time evolution is shown in Fig. 3(A)], for the case $\tilde{\Gamma}>0$, the time evolution is shown as Fig. 3(B). The symbols (A) and (B) indicate the type of the time evolution illustrated as Figs. 3(A) and 3(B), respectively.

	$(1)'$ $0<\phi_2/2<\pi/2$	$(2)'$ $\pi/2<\phi_2/2<\pi$	$(3)'$ $-\pi/2<\phi_2/2<0$	$(4)'$ $-\phi<\phi_2<-\pi/2$
(1) $0<\phi_1/2<\pi/2$	$(\Omega,\tilde{\Gamma})=(+,-)$	$(+,+)$	$(+,+)$	$(+,-)$
	$\lvert N\rvert=\infty,(A)$	$0,(B)$	$0,(B)$	$\infty,(A)$
(2) $\pi/2<\phi_1/2<\pi$	$(-,-)$	$(-,+)$	$(-,+)$	$(-,-)$
	$0,(A)$	$\infty,(B)$	$\infty,(B)$	$0,(A)$
(3) $-\pi/2<\phi_1/2<0$	$(-,-)$	$(-,+)$	$(-,+)$	$(-,-)$
	$0,(A)$	$\infty,(B)$	$\infty,(B)$	$0,(A)$
(4) $-\pi<\phi_1/2<-\pi/2$	$(+,-)$	$(+,+)$	$(+,+)$	$(+,-)$
	$\infty,(A)$	$0,(B)$	$0,(B)$	$\infty,(A)$

$$\cos\frac{\phi_1+\phi_2}{2}=0,\quad \frac{Q}{P}=\frac{\beta}{\delta}=a, \tag{27}$$

and

$$\cos\frac{\phi_1-\phi_2}{2}=0,\quad \frac{Q}{P}=\frac{\beta}{\delta}=a. \tag{28}$$

There are two types of resonant interaction between line soliton and growing-and-decaying mode, which is classified by using the parameters $(\phi_1/2,\phi_2/2)$ as shown in Table I.

At first we investigate the case (1)-$(1)'$ in the Table I: $0<\phi_1/2<\pi/2$ $(\Omega>0)$ and $0<\phi_2/2<\pi/2$ $[\tilde{\Gamma}=\Gamma-(2kP+2lQ)<0]$. In this case, we can take the parameters $(P,Q,\beta,\phi_1,\phi_2)$ close to the plane in the parameter space where the resonant condition is satisfied, i.e., $|N|\to\infty$. We study the time evolution of soliton in the following five periods in time. The solution is approximated in each period as follows:

$(p_1)t\to-\infty$ (before the mode grows)

$$f=\frac{M}{4}e^{2(-\Omega t+\sigma_1)}(1+e^{\xi+\sigma_3}), \tag{29}$$

$$g=\frac{M}{4}e^{2(-\Omega t+\sigma_1+i\phi_1)}(1+e^{\xi+\sigma_3+i\phi_2}), \tag{30}$$

where $\sigma_3=2\ln|N|$, before growing, only the line soliton exists in the wave field as shown in Fig. 1(a).

$$(p_2)t\sim\frac{\sigma_1}{\Omega}[e^{-\Omega t+\sigma_1}\sim O(1)].$$

(a) $s\ll s_0$

$$f\simeq 1+e^{-\Omega t+\sigma_1}\cos\eta+\frac{M}{4}e^{-2\Omega t+2\sigma_1}, \tag{31}$$

$$g\simeq 1+\exp(-\Omega t+\sigma_1+i\phi_1)\cos\eta+\frac{M}{4}\exp(-2\Omega t+2\sigma_1+2i\phi_1). \tag{32}$$

(b) $s\sim s_0$

$$f\simeq 1+e^{-\Omega t+\sigma_1}\cos\eta+\frac{M}{4}e^{-2\Omega t+2\sigma_1}+\frac{M}{4}\exp(\xi+\sigma_3-2\Omega t+2\sigma_1), \tag{33}$$

$$g\simeq 1+\exp(-\Omega t+\sigma_1+i\phi_1)\cos\eta+\frac{M}{4}\exp(-2\Omega t+2\sigma_1+2i\phi_1)+\frac{M}{4}\exp[\xi+\sigma_3-2\Omega t+\sigma_1+i(\phi_2+2\phi_1)]. \tag{34}$$

$$\text{(c)}\quad s_0\ll s \tag{35}$$

$$f\simeq e^{\xi+\sigma_3}\frac{M}{4}e^{-2\Omega t+2\sigma_1}, \tag{36}$$

$$g\simeq\frac{M}{4}\exp(\xi+\sigma_3-2\Omega t+2\sigma_1), \tag{37}$$

where s is the coordinate of propagating direction of the line soliton $(s=x/\sqrt{1+a^2}+ay/\sqrt{1+a^2})$, s_0 is a position of the line soliton. In this period, the mode is growing only in the region $s<s_0$, but the mode has not grown as yet in $s_0<s$ as shown in Fig. 1(b).

$$(p_3)\quad t\sim\frac{\sigma_1+\frac{1}{4}\sigma_3}{\Omega}[e^{-\Omega t+\sigma_1}\ll 1\ \text{ but }\ \sqrt{|N|}e^{-\Omega t+\sigma_1}\sim O(1)].$$

$$f\simeq 1+\frac{M}{4}\exp(\xi+\sigma_3-2\Omega t+2\sigma_1), \tag{38}$$

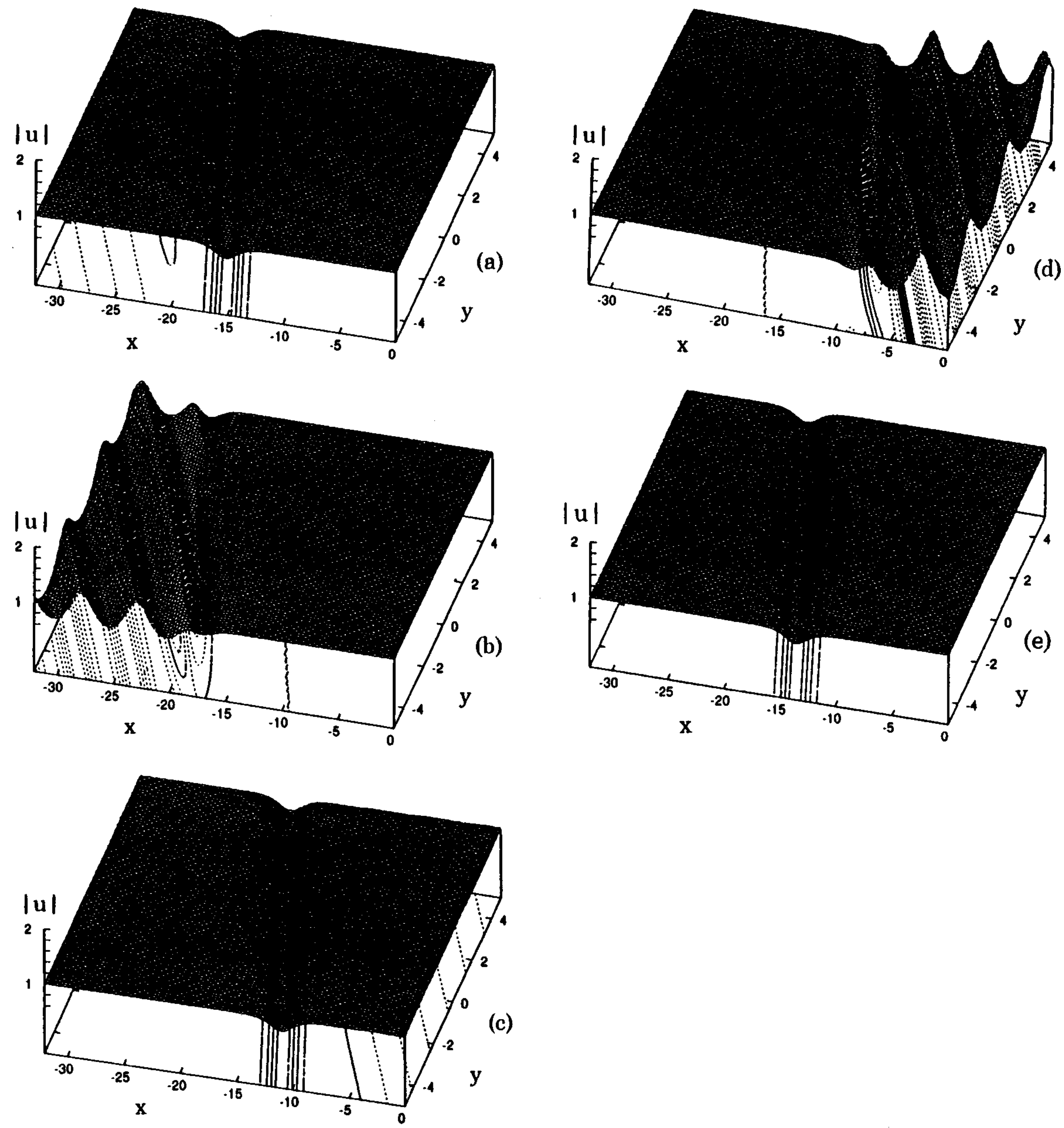

FIG. 1. The sequence of snapshots of the quasiresonant interaction between line soliton and growing-and-decaying mode as illustrated by Fig. 3(A). The parameters are $(k,l)=(0.10,0.10)$, $(P,Q)=(1.403,1.068)$, $(\beta,\delta)=(1.273,1.672)$, and $(\phi_1/2,\phi_2/2)=(5\pi/18,2\pi/9)$. The shots (a) to (e) correspond to the periods (p_1) to (p_5) each to each. (a) $i=-2.0$, (b) $i=0.40$, (c) $t=2.80$, (d) $t=4.0$, (e) $t=7.60$. In this figure x, y, and u are all dimensionless.

$$g \simeq 1+\frac{M}{4}\exp[\xi+\sigma_3-2\Omega t+2\sigma_1+i(\phi_2+2\phi_1)]. \tag{39}$$

This solution describes the line soliton in the resonant state as shown in Fig. 1(c). The mode existed in the region $s<s_0$ has already decayed. From the resonant condition in this case

$$\frac{\phi_1+\phi_2}{2}=\frac{\pi}{2}, \quad \frac{Q}{P}=\frac{\beta}{\delta}=a, \tag{40}$$

we see that

$$\sin^2\frac{\phi_2+2\phi_1}{2}=\sin^2\left(\pi-\frac{\phi_2}{2}\right)=\sin^2\frac{\phi_2}{2}, \tag{41}$$

$$2\Omega+\Gamma-2P(k+al)=-P^2(1+a^2)\cot\frac{\phi_2+2\phi_1}{2}$$
$$=P^2(1+a^2)\cot\frac{\phi_2}{2}. \tag{42}$$

Therefore the resonant line soliton has the same wave number $(P,Q=Pa)$ as the original ones and the value ϕ_2 changes to $\phi_2+2\phi_1$, but the dispersion relations are satisfied as shown in Eqs. (41) and (42). The line soliton has been

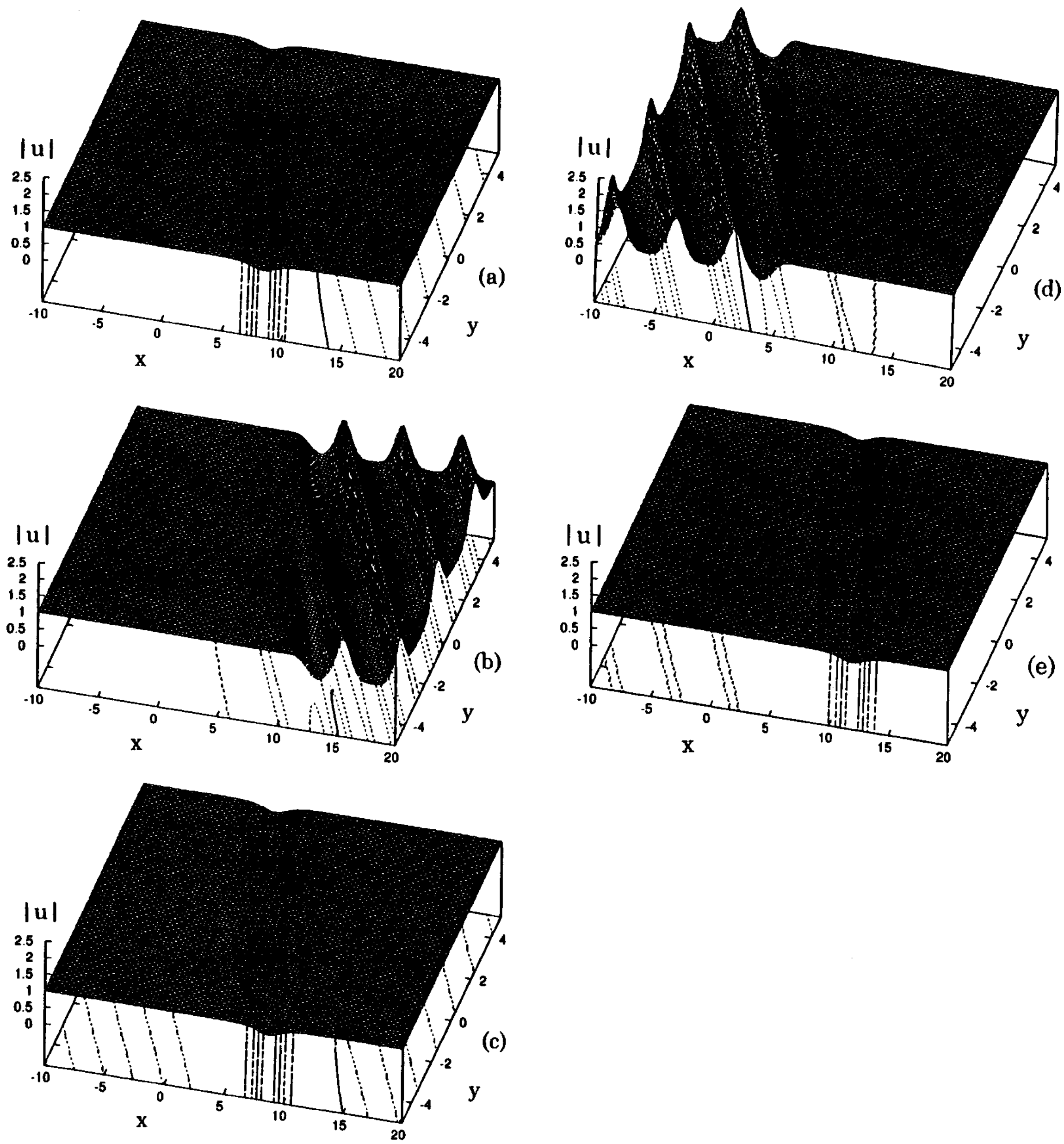

FIG. 2. The sequence of snapshots of the quasiresonant interaction between line soliton and growing-and-decaying mode as illustrated by Fig. 3(B). The parameters are $(k,l)=(0.10,0.10)$, $(P,Q)=(1.403,1.068)$, $(\beta,\delta)=(1.273,1.672)$, and $(\phi_1/2,\phi_2/2)=(5\pi/18,7\pi/9)$. The shots (a)–(e) correspond to the periods (p_1) to (p_5) each to each. (a) $t=-6.30$, (b) $t=-4.30$, (c) $t=-2.30$, (d) $t=-0.30$, (e) $t=0.2.70$. In this figure x, y, and u are all dimensionless.

accelerated as a result of growth and decay of the mode existed in $s<s_0$. The position of the line soliton is given by $\xi+\frac{1}{2}\sigma_3\approx 0$.

$$(p_4)\ t\sim\frac{\sigma_1+\frac{1}{2}\sigma_3}{\Omega}[|N|e^{-\Omega t+\sigma_1}\sim O(1)],\quad \text{for}\quad s_0<s,$$

$$f\simeq e^{\xi}\left(1+N\,e^{-\Omega t+\sigma_1}\cos\,\eta+\frac{M}{4}|N|^2e^{-2\Omega t+2\sigma_1}\right),\quad (43)$$

$$g\simeq e^{\xi+i\phi_2}\left(1+N\exp(-\Omega t+\sigma_1+i\phi_1)\cos\,\eta+\frac{M}{4}|N|^2\right.$$
$$\left.\times\exp(-2\Omega t+2\sigma_1+2i\phi_1)\right),\quad (44)$$

where we have neglected N_i, since $|N_i|\ll 1$. In this period, the mode is developed where only $s_0<s$ as shown in Fig. 1(d).

$$(p_5)\quad t\rightarrow+\infty$$

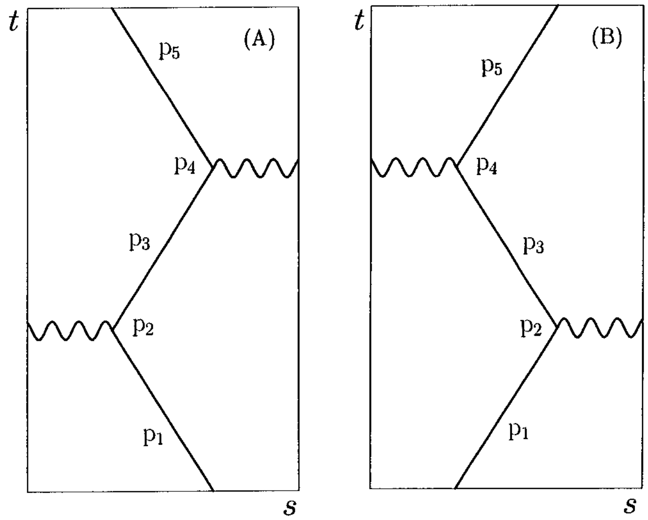

FIG. 3. (A) The schematic diagram of the quasiresonance between line soliton and growing-and-decaying mode in the s-t plane. The solid lines are the line soliton hum and the wavy lines are the growing-and-decaying mode. The marks (p_1) to (p_5) coincide with the periods used in the discussion, respectively. The growing-and-decaying mode begins to grow in the region $s<s_0$ first. (B) The same diagram as (A). The growing-and-decaying mode begins to grow in the region $s>s_0$ first.

$$f=1+e^{\xi}, \tag{45}$$

$$g=1+e^{\xi+i\phi_2}. \tag{46}$$

This solution shows the line soliton after the growth and decay of the mode as shown in Fig. 1(e). The line soliton has gotten the phase shift $-2\ln|N|$ due to the growing-and-decaying mode.

The schematic diagram of this phenomenon in (s,t) plane is shown in Fig. 3(A). The lines in the figure are the world lines of line soliton hump and the wavy lines show the growing-and-decaying mode.

The $|N|\to\infty$ resonances are possible in the regions (1)-$(4)'$, (2)-$(2)'$, (2)-$(3)'$, (3)-$(2)'$, (3)-$(3)'$, (4)-$(1)'$, and (4)-$(4)'$ besides the region (1)-$(1)'$. The diagram in Fig. 3(A) illustrates the time evolution of the quasiresonances in the cases (1)-$(1)'$, (1)-$(4)'$, (4)-$(1)'$, and (4)-$(4)'$ schematically. In a similar fashion, Fig. 3(B) shows the time evolution of the quasiresonance in the regions (2)-$(2)'$, (2)-$(3)'$, (3)-$(2)'$, and (3)-$(3)'$.

Next, we consider the case (1)-$(2)'$; $0<\phi_1/2<\pi/2$ $(\Omega>0)$ and $\pi/2<\phi_2/2<\pi$ $(\tilde{\Gamma}>0)$. In this case, we can take the parameters so as N to go to zero, conditions that are given by

$$\frac{\phi_1-\phi_2}{2}=\frac{\pi}{2}, \quad \frac{Q}{P}=\frac{\beta}{\delta}=a. \tag{47}$$

By the same discussion as the last case, we can study the evolution of the solution. The sequence of snapshots of Fig. 2 shows the evolution of the solution in this case.

The instability develops first in front of the line soliton (in $s_0<s$), as if the line soliton induced the instability. Then the line soliton is decelerated as a result of the instability and the wave field shifts to the intermediate state, a solution that is given by

$$f=\frac{M}{4}e^{-2\Omega t+2\sigma_1}\left(1+\frac{4}{M}e^{\xi+2\Omega t-2\sigma_1}\right), \tag{48}$$

$$g=\frac{M}{4}\exp(-2\Omega t+2\sigma_1+2i\phi_1)\bigg(1+\frac{4}{M}\exp[\xi+2\Omega t-2\sigma_1 + i(\phi_2-2\phi_1)]\bigg). \tag{49}$$

From the condition (47), we see that

$$\sin^2\frac{\phi_2-2\phi_1}{2}=\sin^2\left(\pi-\frac{\phi_2}{2}\right)=\sin^2\frac{\phi_2}{2}, \tag{50}$$

$$\Gamma-2\Omega-2P(k+al)=-P^2(1+a^2)\cot\frac{\phi_2-2\phi_1}{2} = P^2(1+a^2)\cot\frac{\phi_2}{2}, \tag{51}$$

which are the dispersion relations of the line soliton in the intermediate state. This intermediate state lasts about $-\ln|N|$. The mode instability backward the line soliton set up the growth at the end of the intermediate state. Finally, the line soliton is accelerated and returns to the original soliton with the ending of the mode. The time evolution of this quasiresonance is illustrated by the diagram in Fig. 3(B).

The $|N|\to 0$ resonances are possible in the regions (1)-$(3)'$, (2)-$(1)'$, (2)-$(4)'$, (3)-$(1)'$, (3)-$(4)'$, (4)-$(2)'$, and (4)-$(3)'$ besides the region (1)-$(2)'$. The time evolution of the quasiresonances in the cases (2)-$(1)'$, (3)-$(1)'$, (2)-$(4)'$, and (3)-$(4)'$ and in the cases (1)-$(2)'$, (1)-$(3)'$, (4)-$(2)'$, and (4)-$(3)'$ are illustrated schematically by the diagrams in Figs. 3(A) and 3(B), respectively.

III. CONCLUSIONS

We have investigated the time evolution of the quasiresonant interaction between line soliton and growing-and-decaying mode. Under the quasiresonant condition, the mode develops first in the forward region of propagation of the line soliton. The line soliton is accelerated as a result of the growth and decay of the mode existed in the forward region and the wave field shifts to the intermediate state, where the only line soliton exists. This intermediate state persists over a comparatively long time interval. After sufficiently long time, the mode starts to grow in the opposite side of the line soliton. The existence of soliton changes the evolution of the growing-and-decaying mode drastically as if the line soliton dominated the evolution of the instability in the whole region of the wave field.

[1] T. B. Benjamin and J. E. Feir, J. Fluid Mech. **27**, 417 (1967).
[2] J. Feir, Proc. R. Soc. London, Ser. A **299**, 54 (1967).
[3] V. E. Zakharov, J. Appl. Mech. Tech. Phys. **9**, 86 (1968).
[4] D. J. Benny and G. Roskes, Stud. Appl. Math. **48**, 377 (1969).
[5] A. Davey and K. Stewartson, Proc. R. Soc. London, Ser. A **101**, 101 (1974).
[6] N. C. Freeman and A. Davey, Proc. R. Soc. London, Ser. A **344**, 427 (1975).
[7] M. Tajiri and T. Arai, Phys. Rev. E **60**, 2297 (1999).
[8] D. Pelinovsky, Phys. Lett. A **196**, 181 (1994).
[9] Y. Watanabe and M. Tajiri, J. Phys. Soc. Jpn. **67**, 705 (1998).
[10] M. Tajiri, T. Arai, and Y. Watanabe, J. Phys. Soc. Jpn. **67**, 4051 (1998).
[11] T. Arai, K. Takeuchi, and M. Tajiri, J. Phys. Soc. Jpn. (to appear).
[12] J. Satsuma and M. J. Ablowitz, J. Math. Phys. **20**, 1496 (1979).